Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

TRANSACTIONS

OF THE

ROYAL IRISH ACADEMY.

I.—*On the Relation between the Temperature of metallic Conductors, and their Resistance to Electric Currents.* *By the* Rev. THOMAS ROMNEY ROBINSON, *D.D., M.R.I.A., &c. &c.*

Read November 30, 1848.

IN the year 1821, Sir H. Davy discovered that metallic wires resist a voltaic current more as their temperature is raised; and that this is the case whether they be heated by the current itself or some other means. His memoir contains many remarkable facts; but the imperfect state of rheometric knowledge at that time, and the unsteady action of the batteries which were then in use, prevented him from determining the law of the change. More recently the subject has been examined by E. Becquerel and Lenz, who found the increase of resistance proportional to the temperature. Their researches (to which however, I have not an opportunity of referring) were, I believe, made at temperatures little above that of boiling water; and it seemed desirable to extend them through a wider range, as facts of this nature have an unequivocal relation to the molecular forces and atomic structure of matter.

The transference of electricity through a wire has nothing in common with the movement of material fluids in a tube, except the analogy of the effects produced by enlarging the section of the conductor. A much more probable view of its nature is that which refers it to a successive change of tension in each molecule, acting by induction through the interval between them in a way corresponding to the charge and disruptive discharge of a coated plate. This in some degree accounts both for the heat produced by the current, and for its increasing with the resistance. It also seems to explain the increase of resistance; for the distance increases with the expansion, and the intensity required for discharge increases with the distance. That intensity must, by the ordinary theory, be as the square of the distance between the particles; and, therefore, the resistance is in the same ratio. Calling it A and the distance z; if this latter become by expansion $z + \zeta$, the proportional change of resistance

$$\frac{A' - A}{A} = \frac{2\zeta}{z} + \frac{\zeta^2}{z^2};$$

or, if $l + eP$ be the length of the expanded wire l,

$$A' - A = A\left(\frac{2eP}{l} + \frac{e^2P^2}{l^2}\right)$$

It will, therefore, increase faster than the expansion.

Or the transfer may be regarded in another view, as merely a momentary change of condition propagated in rapid succession like a wave, and modifying as it passes the normal action of the forces which keep the molecules in their state of equilibrium. A statement of this kind is of necessity vague, in our present imperfect knowledge (or rather conjectures); yet we are not without grounds for believing, that when these forces exert any specific action, they become less efficient as to others, which, under different circumstances, they would have maintained with energy. Thus, light interferes with affinity, and this with cohesion; thus also, pressure developes heat and electricity. It is, therefore, possible, that while producing thermic effects they may be unable to contribute to electric conduction. If so, we may expect that the change will be proportionate to its cause, to the heat developed in the conductor.

However this may be, whatever tends to throw light on a question so important cannot be without its use; and I hope the experiments which I have the honour to lay before the Academy may in some degree answer this purpose.

The rheometer and rheostat which I employed have been described by me in a previous communication*. To these is added a pyrometer, which measures the temperature by the expansion of the platinum wire which is the subject of experiment. It is shown at figs. 1 and 2. The wire *w*, 5.4 inches long, is held above by a clip attached to the binding-screw *a*, which slides in the piece *b*, and can be pinched in it at any position. Below, the wire is passed round the cylinder *c*, and secured in front by a screw. The arbor *g*, which carries the cylinder, has also an arm *d*, provided with a sectoral arc *e*, grooved on the exterior to lodge the silk thread *f*, attached to its lower extremity, and coiled above on the arbor *i*. This is furnished with a pulley *h*, on which is wound another thread bearing a counterpoise: it also carries the index *n*, which plays on a dial divided into sixty parts. It will easily be understood from this, that the counterpoise tends to pull up the arm *d*, and roll the wire on the cylinder *c*, but is resisted by its tensile force. If now the wire

Fig. 1. Fig. 2.

---

[*] **Transactions** R. I. A. vol. xxi., Science, p. 291.

be elongated by expansion, the counterpoise will descend, and the index n will describe an arc of the dial proportional to that expansion. A strong platinum wire screwed into the lower part of c dips in the mercury-cup r, and by connecting this and the binding-screw a with a battery, any required current is passed through the wire w. Since the mass of the cylinder c, and its arbor, is very great in comparison of that part of the wire which is in contact with it, besides being connected with the metallic frame of the pyrometer, it is scarcely heated; and, therefore, its expansion may be neglected. The effective length of the wire may, therefore, be taken as the distance between the centre of the arbor and the bottom of the clip. Calling this l, the number of the pyrometer divisions P, and the value of one of them (in units of l) $= e$, the machine gives eP equal to the expansion—not of l,—but of a portion of it l', whose length, *when expanded*, is equal to l. We have, therefore,

$$l' = l - eP,$$

and obtain the expansion of l itself by the equation

$$\epsilon l = \frac{eP \times l}{l - eP} = eP\left(1 + \frac{eP}{l} + \&\text{c.}\right)$$

In the instrument which I constructed, the length of $d = 3^{\text{i}}.00$; the radius of $c = 0^{\text{i}}.208$; and the diameters of the arbor i and the dial are respectively $0^{\text{i}}.1783$ and $3^{\text{i}}.24$. As each of the divisions $= 0^{\text{i}}.17$ nearly, the value of e is $0^{\text{i}}.17 \times \frac{0.208}{3.00} \times \frac{0.18}{3.24}$, or $0^{\text{i}}.00065$; and tenths of this are easily estimated. It was more exactly determined by lowering the clip an amount measured by a micrometer microscope, which gives as a mean of eighteen trials corresponding to 147^{P}, 0.09516, or,

$$e = 0.000643.$$

The diameter of the counterpoise-pulley $= 0^{\text{i}}.437$, and, therefore, any weight applied there causes a tension of the wire 35.3 times as great. The counterpoise consists of a weight $m = 31.3$ grains, which equilibrates the arm d and its sector; and of a piece of chain o which gives the tension. The use of this arrangement is, that when the wire is heated, and unable to bear much strain, the chain descends and rests on the bottom of the pyrometer, so that as it approaches the fusing point of platinum the tension has almost ceased. The front and back of the instrument are brass ; the sides are glazed, but, except in photometric experiments, the glass is covered inside with slips of bright tin, to lessen the effect of radiation. The top is mahogany, to insulate the screw *a*; from its low conducting power this becomes very hot, and therefore exerts a cooling power on the wire much less than what acts at its other end.

To deduce the temperature from the corrected expansion, I have used the expansibility of platinum given by Dulong and Petit.* They assign

$$\text{Mean absolute dilatation from } 0^\circ \text{ to } 100^\circ \text{ Cent.} = \frac{1}{37700},$$

$$\ldots\ldots\ldots \text{ from } 0^\circ \text{ to } 300^\circ \text{ Cent.} = \frac{1}{36300}.$$

The corresponding temperatures, being measured by an air thermometer, might require a slight correction for the coefficient of gaseous expansion, which was Gay Lussac's; but such refinement is needless in the present research. The expansion-rate of the metal evidently increases with the temperature; its law is unknown, but we shall probably not err far by assuming

$$\epsilon = \alpha . t + \beta . t^2,$$

and the above values give

$$0.0000088418 = \alpha \times 180^\circ + \beta \times (180^\circ)^2,$$
$$0.0000091828 = \alpha \times 180^\circ + 3\beta \times (180^\circ)^2;$$

the degrees being Fahrenheit's, but their origin at the freezing point of water. Hence

$$\alpha = \log^{-1}(4.68282); \ \beta = \log^{-1}(0.72118).$$

But since ϵ is the absolute increase, divided by the length, we have

$$\frac{eP}{a . l} = t\,(1 + ft) = \eta P,$$

f being $\frac{\beta}{a} = \log^{-1}(6.03836)$, and $\eta = \frac{e}{al}$.

This quadratic may be solved in each experiment, or its positive root tabu-

* Annals of Philosophy, 1819.

lated for a decimal progression of P, and the intermediate values got by interpolation.

As these temperatures are reckoned from 32°, but the reading sets out from the actual temperature of the atmosphere, a correction is applied by adding to it the required amount.

When the heat was expected to be powerful, an additional resistance, sometimes as much as 1500^R, was included in the circuit, and gradually lessened till the full current passed. If this precaution be not attended to, the momentum which the counterpoise, &c., acquires in its rapid descent, is sufficient to produce a permanent elongation of the wire in its softened state, so that the index will not return to zero.

The battery used was at first on Daniell's principle, the acting surface of each metal being fifty-four square inches. With small wires this acts very well; but I found that, when the resistance of the circuit is little, the sulphate of copper is expended more rapidly than it can be supplied. In this respect, as also in giving a more powerful current, the chloride of copper is a better charge. Afterwards I used Grove's cells, each having 19.3 inches of platinum, and found them much more convenient. When the negative charge is 2 nitric acid, 2 water, and 1 sulphuric acid, they exceed in power twice as many of the others, and for a long time there is no extrication of nitric oxide. All inconvenience from this may be avoided by arranging them outside the window, and bringing the conductors through its wood-work.

When the circuit is completed through the pyrometer, its index moves very rapidly at first; several seconds, however, elapse before the wire becomes luminous, and $30^{s.}$ or $35^{s.}$ are necessary for its attaining its full heat: when it becomes stationary, a few light taps are given to the stand, to loosen any friction of the pivots. The ignition never extends to the extremities of the wire, especially the lower one; and the upper part of the wire is evidently the hottest, both for the cause already noticed, and the ascending current of heated air. It is a curious circumstance, that, when the temperature is above zero, the wire, which is then dazzling white, seems enlarged to three or four times its real diameter, an effect of irradiation which disappears when it is viewed through a darkly coloured glass. By reducing the current, and with it the intensity of the ignition, the dark portion at the bottom of the wire extends, the upper one remaining nearly as before. If the room be completely darkened, this proceeds till only the upper inch of it is visible, and the *least additional* decrease of ignition makes all disappear, when the pyrometer shows about 550°.

The first used was originally $\frac{1}{104}$ of an inch diameter; but during the numerous preliminary experiments it was stretched till its thickness was only $\frac{1}{138}$: the results obtained are arranged in the following table, each being a mean of ten trials:

No.	Battery.	ϕ.	Current.	P.	T.	A.	REMARKS.
1	One D.	40°.8	0.809	12.1	289.7	305.5	
2	One G weak.	42°.0	0.858	13.4	319.9	320.8	
3	Two D.	44°.1	0.957	24.1	559.3	385.2	
4	Three D.	48°.2	1.168	41.6	930.9	497.9	
5	Four D.	49°.0	1.214	45.9	1020.6	523.8	Ignition visible in full day.
6	Five D.	51°.3	1.357	55.7	1215.0	591.6	Ignition strong red.
7	Seven D. + ρ.	52°.0	1.404	62.1	1338.7	643.8	48 resistance added.
8	Six D.	53°.7	1.515	67.8	1447.0	666.4	
9	Eight D. + ρ.	54°.0	1.538	73.1	1545.6	706.2	78.7 resistance added.
10	Seven D.	57°.0	1.794	85.0	1761.9	737.4	Yellow heat.
11	Eight D.	59°.0	2.089	96.6	1966.4	793.9	Almost white.
12	Six G.	60°.0	2.200	105.4	2116.0	840.5	Observed at night and doubtful.
13	Nine D.	60°.2	2.222	108.4	2166.2	827.8	
14	Twelve D.	61°.7	2.318	126.4	2461.6	898.2	Pure white and dazzling.

The column headed ϕ contains the deflection of the rheometer; the next gives the intensity of the current, its unit being that which deflects the rheometer to 45°, and disengages in a voltameter 6.57 inches of gases, at their normal temperature and pressure, in five minutes. The two next columns give P, the reading of the pyrometer corrected for the temperature of the air, and T, the temperature of the wire computed from it. The last gives A, the resistance of the wire expressed in revolutions of the rheostat.

The great increase of the resistance to more than four times its original value, and its steady progress at such high temperatures, are very remarkable.

The pyrometer was then placed in vacuo for a purpose that shall be subsequently noticed. As I feared that the sudden change of temperature might fracture a glass receiver, a box of strong copper was used, the lid of which was easily made air-tight by Whitworth's scraping process. Two apertures glazed with strong plate glass enabled me to read the index and inspect the wire; and an insulated wire passing through the top, and dipping in a mercury cup formed in the binding-screw *a*, connected it with the battery. The box was connected by a screw with the air-pump, which, however, was at the time not in good order. The following results are also each a mean of ten:

No.	Battery.	Pressure.	ϕ.	Current.	*P*.	*T*.	*A*.	Remarks.
15	One G. weak.	$0^{i}.64$	41°.2	0.825	22.7	531.4	390.1	Charge had been used.
16	One G.	$0^{i}.42$	43°.3	0.916	32.3	738.7	438.8	
17	Two G. weak.	$0^{i}.60$	45°.5	1.025	45.0	1002.3	515.4	
18	Two G.	$0^{i}.40$	50°.9	1.334	62.5	1346.4	606.2	
19	Three G. weak.	$0^{i}.51$	. . .	. . .	69.6	1480.7	674.5	N. charge, 1 nit. acid + 1 water.
20	Three G.	$0^{i}.40$	56°.0	1.716	89.4	1839.9	750.8	
21	Four G.	$0^{i}.46$	57°.2	1.898	105.3	2116.0	829.5	
22	Five G.	$0^{i}.47$	60°.8	2.211	123.2	2410.4	892.5	

The currents required to produce a given temperature are less, but the resistance is the same; its increase is therefore not due to any intrinsic quality of the current.

On attempting to pass the current of six Groves the wire gave way.

Another piece was substituted, to try whether feeble currents were similarly resisted, its diameter being $\frac{1}{104}$. On passing a current 0.279, the lowest which the actual position of the rheometer permitted me to measure, the wire was heated 24°, and its resistance = 189.4. Increasing the current to 0.315, the wire was heated 9° more, and the resistance became 195.4. In fact, I believe it is impossible to pass any current whatever without changing the resistance in some degree; and think it highly probable, that this has given rise to the opinion entertained by some philosophers, that the resistance is a function of the current.

This wire failed also in an attempt to obtain higher temperatures,* and was replaced by one $\frac{1}{49}$ diameter. The counterpoise was changed till, after several trials, it was found that a tension of four ounces at zero of the pyrometer, was sufficient. I was surprised to find that a given battery produced nearly the same ignition in this as in the smaller wire, the increased current compensating the greater mass; but, from the greater quantity of heat evolved, the upper part of the pyrometer became very hot, so that the clip which holds the top of the wire was blued. This apparently made the resistance greater than the truth; but is not likely to have affected its changes, as the experiments were made in the inverse order of the table, so that the highest temperature was obtained first. After five results the wire was broken by an accident close to the cylinder; and as I had no more of that diameter, I rejoined it by a loop, which, being beyond the part that is ignited, might be expected not to interfere with the temperature produced.

---

* In the first trial the zero of pyrometer was, before contact, -7.2; $P' = 148.8$; zero after, $+25.2$. If the whole of the lengthening took place during the cooling, $P = 159.9$ and $T = 2981.3$. The resistance = 967.1. In a second trial with an additional resistance of 50 in circuit, zero

Each result is a mean of five.

No.	Battery.	ϕ.	Current.	*P.*	*T.*	*A.*	REMARKS.
23	One G. + 83.	60°.0	2.200	15.1	359.0	107.3	
24	. . . . +50.	62°.7	2.437	20.3	476.8	107.5	
25	One G.	67°.9	3.261	31.9	730.2	116.4	
26	Two G. + 50.	70°.1	3.768	46.4	1030.9	135.0	Not visible in strong day-light.
27	Two G.	72°.5	4.824	60.4	1306.5	151.0	Full red.
28	Four G.+105.	73°.9	4.827	70.3	1493.8	173.2	
29	. . . . +48.	75°.9	5.654	86.9	1795.5	189.1	Yellow heat.
30	Four G.	77°.1	6.272	99.3	2012.2	188.1	White heat.
31	Six G.+20.	77°.7	6.588	106.5	2134.6	196.0	
32	Six G.	79°.0	7.286	121.9	2389.4	201.9	Splendid *in bright sunshine.*

The increase is evident here also, although the discordances are greater than in the last series.

Lastly, a wire of $\frac{1}{69}$ was used. At No. 40 the pyrometer was immersed in diluted alcohol, to ascertain whether the resistance increased with the current when the wire was kept cool. The fluid boiled round it with a sharp

---

$= -4.6$; $P' = 140.0$; zero after, $+24.0$. On the same supposition, $P = 148.1$; $T = 2803.0$; and the resistance $= 953.0$. In the third trial it gave way; the ends, however, were not fused, though a very little more would certainly have melted it. The temperature of the centre of the wire in the first of these experiments must have exceeded 3200°.

snapping, and its refraction may make the reading doubtful one or two tenths of a division. All but the last are means of five; that only of two, which, however, agree tolerably.

No.	Battery.	ϕ.	Current.	*P*.	*T*.	*A*.	Remarks.
33	One double G. +155.	56°.1	1.522	15.4	365.9	127.8	
34	. . . . . . +88.	58°.4	2.020	20.5	481.3	144.9	
35	. . . . . . +18.	61°.9	2.411	31.4	719.6	169.0	
36	Two G. + 22.6.	65°.1	2.758	46.0	1020.7	212.0	
37	Three G. + 80.	68°.1	3.226	60.6	1310.4	238.7	
38	. . . +37.6.	69°.8	3.699	71.6	1518.0	270.1	
39	Four G.	72°.9	4.352	91.9	1883.9	318.6	
40	Five G.	81°.3	9.548	8.6	207.8	93.8	
41	Ditto.	73°.9	4.821	110.8	2206.7	346.0	Battery nearly exhausted and zero unsteady.
42	Six G.	74°.9	5.230	119.8	2355.1	351.4	

In examining these tables it is evident,

1. That the increased resistance is not occasioned by any condensation of the current. In No. 40 it passes more easily than in No. 33, though of six-fold power, and able to fuse a far thicker wire in air. It is also worth notice, that in these two cases the *productions* of heat are as 29 : 1; and, therefore, it is not by the mere employment of molecular forces in the production of that agent, but by its accumulating and becoming sensible, that conduction is impeded.

2. The magnitude of the change prevents me from attributing it to a mere change of molecular distance. On this hypothesis we have seen that it will be

$$\frac{A'-A}{A}=\frac{2eP}{l}+\frac{e^2P^2}{l^2}.$$

Now in No. 14, the highest of the set, $\frac{eP}{l}=0.1$, which will give $\frac{A'-A}{A}=0.30$, whereas it really $= 4.68$.

3. Nor is it proportional to the expansion: up to a certain point it may be expressed by the formula $A=a+bP$, but less accurately than by making it depend on the temperature; and

4. It appears to be correctly determined by the equation

$$A=a+bT; \qquad (1)$$

where a is the resistance at 32° of Fahrenheit, and b its change for one degree.

Before endeavouring to deduce the values of these constants for each wire, it is to be remarked, that the resistances given in the preceding tables are too small, and require to be corrected for the heating effect of the current on the rheostat and resistance coils by which they were measured. The thickness of the wire in the former, and the immersion of the others in alcohol, might seem to guard against this danger ; but, with powerful currents, both become warm to the touch. If we assume the truth of (1), the resistance measured is not A, but $A\ (1+b't')$, supposing all reduced to the freezing point. Now the heat generated is as $A \times$ square of current ; and I have found by experiment, that the temperature of a wire follows the same law under 100° Hence, for $1+b't'$, we may write $1+c.A.C^2$, and (1) becomes

$$A = a + bT - c.A^2.C^2.$$

Each result furnishes an equation of condition, which may be grouped together, and either by minimum squares or ordinary elimination the values of a, b, and c determined.

If the twenty-two that belong to the wire $\frac{1}{138}$ be thus combined, we have the equations

$$442.19 = a + b \times 674.1 - c \times 195855.0,$$
$$660.53 = a + b \times 1447.9 - c \times 1002800.0,$$
$$833.32 = a + b \times 2153.8 - c \times 3141477.0.$$

Hence,

$$a = 198.4\ ;\ b = 0.3412\ ;\ c = 0.00003182.$$

Computing with these the apparent values of A, and subtracting them from the observed, I have arranged the results according to T.

No.	*T.*	Obs. – Cal.	No.	*T.*	Obs. – Cal.	No.	*T.*	Obs. – Cal.
1	289.7	+10,2	6	1215.0	+ 4.0	11	1966.4	+10.5
2	320.0	+15.7	7	1338.7	+15.3	12	2116.0	+28.9
15	531.4	+13.6	18	1346.4	-30.8	21	2116.0	-12.0
3	559.3	+ 8.0	8	1447.0	+ 4.1	13	2166.2	- 2.0
16	738.7	+ 7.0	19	1480.7	- 3.3	22	2410.4	- 4.4
4	930.9	+ 6.8	9	1545.6	+18.0	14	2461.6	- 2.1
17	1002.3	+ 2.3	10	1761.9	- 1.0			
5	1020.6	+ 5.5	20	1839.9	-11.3			

With the exception of 18 and 12, the agreement between the formula and observation is sufficiently close; and even in them the error is not remarkable, if we consider that a degree of the pyrometer represents 20° of Fahrenheit, and that a little oxidation in the connexions may affect the resistance of an entire set. It even seems to me that this principle affords a very effectual method of measuring high temperatures in the arts.

With the wire $\frac{1}{49}$ the three equations are

$$\begin{aligned} 114.55 &= a + b \times 424.25 - c \times 130725.0, \\ 171.10 &= a + b \times 1531.93 - c \times 753848.0, \\ 195.33 &= a + b \times 2178.70 - c \times 1741069.0. \end{aligned}$$

Hence,

$$a = 91.4\ ;\ b = 0.05898\ ;\ c = 0.0000141.$$

The value of a is certainly too large. I have mentioned the probable effect of the oxidation of the clip; possibly while looping the broken wire, the contact may have been improved, for I found afterwards, that by trying its resistance under water while attached to the clip, with a current = 2.743, the resistance was 76.1. On making the contact perfect it was only 46.

The values of observed – calculated resistances are

No.	*T.*	Obs. – Cal.	No.	*T.*	Obs. – Cal.
23	359	– 4.5	28	1494	+ 3.6
24	477	–11.1	29	1796	+ 7.9
25	730	–16.1	30	2012	– 2.4
26	1031	–13.5	31	2135	+ 2.2
27	1306	–11.6	32	2389	+ 0.1

Part of the errors of the first five are, I think, accounted for; and the rest is unimportant.

Lastly, for the wire $\frac{1}{69}$ the equations are

$$\begin{aligned} 147.22 &= a + b \times 522.27 - c \times 96511, \\ 240.27 &= a + b \times 1283.03 - c \times 644327, \\ 338.67 &= a + b \times 2148.57 - c \times 2694208. \end{aligned}$$

Hence,

$$a = 81.9\ ;\ b = 0.1261;\ c = 0.000007188.$$

In these equations No. 40 is not included from the uncertainty of P. The comparison with observation gives,

No.	T.	Obs. − Cal.	No.	T.	Obs. − Cal.
33	366	0.0	38	1518	+ 2.0
34	481	+ 3.2	39	1884	+ 9.3
35	720	− 2.7	40	209	−10.1
36	1021	+ 3.2	41	2207	+ 0.5
37	1310	− 5.3	42	2355	− 9.7

Here also the conformity to observation is very satisfactory.

If we divide the values of b by a (assuming $a'' = 46$), we obtain 0.0017, 0.0013, and 0.0015, taking the mean of which we have for a platinum wire,

$$A = a\,(1 + 0.0015 \times T), \qquad (2)$$

a being, as before, the resistance at 32°.

It would be desirable to extend these experiments to other metals ; but I could not find any determination of their expansions similar to that which Dulong and Petit have given for platinum, except what those philosophers have given for iron and copper. The first of these metals, however, I found to oxidate so rapidly in the pyrometer, that I could get no consistent results. It should for this purpose be surrounded by dry nitrogen. Copper also oxidates; but the film of oxide acts as a coating, and protects the interior, so that its diameter does not change up to 900°. The wire used was of the same diameter as the last platinum one, $\frac{1}{69}$. Each result is a mean of three closely agreeing.

No.	Battery.	ϕ.	Current.	P.	T.	A.	Obs. − Cal.
43	1 G. + 196	56°.9	1.808	11.1	102.3	30.1	+ 0.3
44	. . . + 105	64°.3	2.640	21.6	195.0	33.4	+ 0.3
45	. . . + 50	70°.6	3.902	40.0	348.9	37.8	− 0.6
46	. . . + 30	72°.9	4.352	53.0	452.4	41.8	− 0.2
47	1 G.	75°.8	5.608	81.0	664.1	49.6	+ 0.2

When the current was increased to produce ignition, $T = 919.8$, $A = 68.8$; but the zero changed, and the wire stretched till its diameter became $\frac{1}{92}$.

For this metal the coefficients of expansion are $\alpha = \log^{-1}(5.03826)$; $\beta = \log^{-1}(1.40598)$. As the resistance was measured by the rheostat alone, the correction cA^2C^2 need not be applied, and the equations are

$$33.77 = a + b \times 215.40,$$
$$45.90 = a + b \times 558.25.$$

Hence,

$$a = 26.37;\ b = 0.0348;\ \frac{b}{a} = 0.00133.$$

The value of $\frac{b}{a}$ is so near that of platinum as to make it an object of interest to ascertain whether the same equality prevails in other metals. The difference of conducting power in copper and platinum appears very strikingly here. At 32° the resistances are as 1 : 3.1; but in Nos. 46 and 39, when the current is the same, as 1 : 7.6. It may also be remarked, that these constants give, as the probable values of the correction cA^2C^2, quantities closely agreeing with those computed by the values of c given above.

The facility of bringing the wire of the pyrometer to a given temperature, and maintaining it, makes it a convenient source of light in photometers; but as the heat is not uniform along it, it seems worth inquiry according to what law it varies. Each section of the wire is traversed by the same current, and, therefore, under similar circumstances, would be equally heated: but the heat thus excited is dissipated by three cooling agencies. The first of these is radiation, which, though lessened by the bright metallic surface of the pyrometer, below what it would be in free space, is still very powerful. The second is the presence of air; and the third the conducting power of the wire itself, by which a portion of the heat escapes to the metallic supports which attach it to the instrument, in this case chiefly to the lower one. As long as their combined effects are inferior to the heating power of the current, the temperature must increase: while doing so, however, the resistance also increases, and with it (as shall be immediately shown) the heating power, the current being the same. On the other hand, the cooling causes also augment in energy, and in a still higher ratio. An equilibrium of these powers is therefore attained, to which belongs the thermic state shown by the pyrometer: it is, therefore, expressed by equating to cypher the differential equation, which expresses the rate of cooling for a differential section of the wire in function of the time. Calling x the distance of a point from the extremity of the wire, y the excess of its temperature above the surrounding medium, and $f(y)$, $f'(y)$, the heating power of the current, and that cooling one which depends on its surface, and on the surrounding media, the differential equation becomes

$$0 = H.f(y) - I.f'(y) - \frac{K.d^2y}{dx^2}.$$

The integral of which will determine y the temperature at x.

The heating power of a given voltaic current is known to be proportional to the resistance which it overcomes; but in all the experiments which have established these laws, the change of resistance which I have been considering was overlooked, because the conductors were kept at a comparatively low temperature. It might, therefore, be a question, whether the resistance to be used is the intrinsic (that at a given temperature) or that increased by heat. It is easily proved to be the latter by means of the apparatus (fig. 3). A is a thin jar, in whose neck are cemented copper wires terminating in the binding screws C, C. Their other ends are connected by platinum wire, W, of $\frac{1}{69}$ inch diameter, and 5.4 inches long. Over the wire is inverted another jar, B, formed of thin tube. If water be now poured in, B acts as a diving-bell, and the wire W is in contact with air. Passing a current through it, it may be intensely heated, and its resistance of course increased; but the heat which it gives off is employed in heating the glass and water by which it is surrounded, and can be measured by a thermometer immersed in the water. At low temperatures Newton's law of cooling is exact, and, therefore, the rise of the thermometer is proportional to the thermic power of the current. A single result will be sufficient. The jar

Fig. 3.

containing 5.5 cubic inches of water, a current = 3.527 was passed for twelve minutes, the air and water being both nearly 72°. The thermometer rose 77°.5, and the resistance A = 257.6. The wire was almost white, and its temperature must have been near 1500°. The instrument was then cooled, filled with water (which, by inclining it, was made to fill B), a resistance = 165.6 added in the circuit, and a current = 3.558 passed for twelve minutes. Now the thermometer rose only 29°.7, and the resistance = 89°.0. Here all was the same, except the increase of A by the heat; and that determined the greater heating power in the first experiment. The second was repeated, but without any interposed resistance in the circuit: bubbles of steam formed round the wire, which were condensed with sharp snapping; the current = 6.045, which would have melted it in air; the resistance = 94.5; and the rise of the thermometer = 83°.2.

From this it follows that $f(y) = a(1 + ry)$.

The function $f'(y)$ is far from being so easily determined. Fourier and Poisson, in their celebrated investigations, have assumed it $= ky$, following Newton. This, however, is quite at variance with observation. Dulong and Petit, in their memoir, have assigned expressions for the effect of the air and radiation, which represent their observations very exactly. According to them the effect of radiation is as $1.0077^{T} - 1$, and that of the air as $T^{1.23}$. The heated body which they employed was the bulb of a thermometer enclosed in a globe of copper a foot in diameter, blackened on its interior surface, and kept at an invariable temperature. The highest range was under 500°, and of course far below the point at which light is given off,—an agent which, doubtless, interferes with heat. Accordingly, their law of radiation fails altogether in my experiments, but the other is nearly exact. We are enabled to infer this from the law already mentioned, that the heating power of the current is as its square multiplied by the resistance.* Now in the experiments Nos. 1–14, the wire is in contact with air, while in Nos. 15–22, which were made *in vacuo*, the effect of that medium is nearly insensible. If from the first we deduce by interpolation the currents and resistances which correspond to the temperature of the vacuum series, we obtain one in which radiation and conduction must be the same as in it; and the difference between the AC^2 for the same temperature is evidently the measure of the cooling due to air. Comparing them with the corresponding values of T, I find that they are as its first powers. The deficiency from the experiment of Dulong and Petit proceeds, probably, from the air being heated.

---

* This law has been often verified, but the experiments just described gave a good illustration of it. In the first and third, the ratio of the heats excited is 1 : 1.074; while that of AC^2 is 1 : 1.077; the difference arising from this, that, the temperature being higher in the last, more was lost. It may also be mentioned as evidence of the heating power of that current, that it generated in the wire as much heat as would have ignited eight ounces of platinum to strong redness.

In the vacuum series AC^2 must be as the combined effects of radiation and conduction. Omitting No. 15 as too low, if we divide AC^2 by T^2, we obtain the numbers

No. 22, ...	0.75
21, ...	0.67
20, ...	0.66
19, ...	0.64
18, ...	0.73
17, ...	0.54
16, ...	0.68

And if we allow for the residual air and the conduction, we may assume the radiation in this pyrometer to be as the square of the temperature. Hence, $f'(y) = Gy + I.y^2$, and as G is less than $H.ar$, the equation becomes,

$$\frac{d^2y}{dx^2} = my^2 - ny - p,$$

in which it must be remembered that m, the quotient of the coefficient of radiation by that of conduction, is constant for a given wire, but n and p vary with the current. The integration of this is facilitated by considering, that the effect of conduction must cease at a certain distance from the origin, beyond which y is constant. Let this value of y be θ, then we must have,

$$m\theta^2 - n\theta = p, \qquad (3)$$

substituting which, and writing μ^2 for $2m\theta - n$, and u for $\theta - y$, we have,

$$\frac{d^2u}{dx^2} = u\,(\mu^2 - mu).$$

Integrating

$$\left(\frac{du}{dx}\right)^2 = u^2\left(\mu^2 - \tfrac{2}{3}mu\right),$$

which requires no arbitrary constant, because du vanishes with u.

Integrating again, e being the base of the Neperian logarithms,

$$u = \frac{6\mu^2}{m} \times \frac{1}{\left(e^{\frac{\mu(x+\kappa)}{2}} + e^{-\mu\frac{(x+\kappa)}{2}}\right)^2}.$$

As y vanishes when $x = 0$, we have then

$$\theta = \frac{6\mu^2}{m} \times \frac{1}{\left(e^{\frac{\mu\kappa}{2}} + e^{-\frac{\mu\kappa}{2}}\right)^2}, \tag{4}$$

and hence deduce

$$\frac{y}{\theta} = 1 - \frac{(1 + e^{-\mu\kappa})^2}{e^{\mu x}\,(1 + e^{-\mu(\kappa+x)})^2}, \tag{5}$$

which determines the thermic condition of any point x.

As the pyrometer gives only T, the mean temperature, we must find its expression for any length x:

$$T = \frac{\int y\,dx}{x} = \theta + \frac{6\mu}{mx} \times \left(\frac{1}{e^{\mu(x+\kappa)} + 1}\right) + \kappa'.$$

In strictness this should be taken from 0 to the centre of that portion of the wire which is of uniform heat, supposing both supports to cool it equally; but as this part extends close to the upper support, I prefer taking it from 0 to λ, the place where the heat declines; in this instrument λ is five inches. Thence

$$\frac{T}{\theta} = 1 - \frac{1}{\mu\lambda}\left(1 + e^{-\mu\kappa} - \frac{(1 + e^{-\mu\kappa})^2}{e^{\mu\lambda} + e^{-\mu\kappa}}\right). \tag{6}$$

It then remains only to determine θ, μ, and $e^{-\mu\kappa}$.

In (3) the quantity n is the difference of two quantities; one the heating power due to the increase of resistance by heat, which, being proportional to p, may be called rp, r being, as we have seen, 0.0015; the other representing the air's cooling power, which, like m, is constant, and therefore $= ms$. Equation (3) then becomes

$$m\theta^2 - \theta\,(rp - ms) = p,$$

which, combined with $\mu^2 = 2m\theta - n$, gives

$$\mu^2 = m\left(\frac{s + 2\theta + r\theta^2}{1 + r\theta}\right).$$

From the series in air and *in vacuo* already referred to, it appears that $s =$ 0.2822; it may, therefore, be neglected, and then

$$\mu^2 = 2m\theta\left(\frac{1 + \frac{1}{2}r\theta}{1 + r\theta}\right). \qquad (7)$$

Substituting this in (4), we obtain

$$e^{\pm\mu x} = \frac{5 + 2r\theta}{1 + r\theta} \pm \sqrt{\left\{\left(\frac{5 + 2r\theta}{1 + r\theta}\right)^2 - 1\right\}}. \qquad (8)$$

Whence it appears, that though μ and θ increase, $e^{-\mu x}$ is confined in narrow limits, ranging from 0.105 to 0.268, while θ passes from 0 to infinity.

Since the loss of heat must equal its production, we have $m\theta^2 + sm\theta$ as AC^2, or,

$$\theta^2 + s\theta = qAC^2,$$

and as s is small, $\theta = \sqrt{(qAC^2)}$. Hence the highest temperature attained is as the current, not as its square; and also as the square root of the resistance.

If we tabulate an equidistant series of θ, and compute for each the values of $e^{-\mu x}$; $\frac{\mu^2}{2m} = f$, and $\frac{\theta \times (1 + e^{-\mu x}) \times \sqrt{(2m)}}{\mu} = g$; and assume $\nu = \lambda \times \sqrt{(2m)}$, the equation (6) gives

$$\theta - T = \frac{g}{\nu}\left(1 - \frac{1 + e^{-\mu x}}{e^{\sqrt{(f)} \times \nu} + e^{-\mu x}}\right).$$

Now taking any two observed values of T, where A and C are known, we have the ratio of the θs; assuming one the other is known, and the other quantities can be taken from the table. A few trials of this kind show that

ν must be a number less than unity, and that $e^{\sqrt{(f)}\times\nu}$ must be so large that the last term of the equation may be neglected, whence

$$\theta - T = \frac{g}{\nu},$$
$$\mu = \frac{\nu}{\lambda} \times \sqrt{f}. \qquad (9)$$

When ν is known θ is easily found; for, entering the table with T as argument, we get a first approximation, with which as argument the true value is obtained.

A still more accurate mode of obtaining this quantity is by observing the value of x at which the wire assumes a given temperature. That which I selected is the point at which platinum begins to be visible in total darkness. It is unknown; but by varying the current, so as to have different lengths of x, we may equate the values of y in (5). I measured x by a screen moved by a rack, which was lowered till it cut off all light. Its pinion was moved by a graduated head, and I found the measures very consistent.

With the wire $\frac{1}{69}$ I obtained

$T = 539.3$; $C = 2.213$; $A = 150.9$; no light visible.
$T' = 772.9$; $x = 1^{i}.29$; $C = 2.375$; $A = 180.9$;
$T'' = 850.5$; $x = 1^{i}.03$; $C = 2.516$; $A = 190.9$.

From these I find $\nu = 0.3$ nearly, and with this value

$$\theta = 710.9; \quad \frac{\theta}{R} = 26.2.$$

$$\theta' = 911.5; \quad \frac{\theta'}{R'} = 28.5; \quad y' = 739.7.$$

$$\theta'' = 995.5; \quad \frac{\theta''}{R''} = 28.6; \quad y'' = 733.2.$$

R being $= \sqrt{A} \times C$.

The light of the wire is like Herschel's lavender ray, and is, perhaps, rather a phosphorescence than a true ignition.

For the wire $\frac{1}{138}$ the same process gives $\nu = 0.4$ nearly, as might be expected from its being proportional to $\sqrt{m}$; the temperature is also something higher, probably from the smaller quantity of light requiring a greater intensity to be visible.

I will conclude with a summary of the principal facts which I have endeavoured to establish in this memoir:—

1. When a wire of platinum is heated by a voltaic current, its resistance to the passage of that current increases without limit to the verge of its fusion.

2. That increase of resistance is not caused by the mere increase of the current.

3. It is not caused by the increased distance of the molecules.

4. It is not caused by the employment of the molecular forces in *generating* heat.

5. It is exactly proportional to the increase of temperature of the wire.

6. The same is the case with copper till the oxidation of the metal interrupts the experiment.

7. In both those metals a given elevation of temperature produces the same proportionate change of resistanee.

8. This change of resistance must always be attended to in rheometry; and the neglect of this precaution may explain some objections that have recently been made to the theory of Ohm.

9. The heat evolved by a current passing through a wire is as the square of the current, and as the actual resistance of the wire (that increased by the heat).

10. The highest temperature attained by any part of it is, however, as the current × square root of resistance.

11. The loss of heat by the air is as the difference of the temperatures of the air and wire.

12. That by radiation is in this pyrometer as the square of that difference nearly.

13. Thermic equation of the wire; from which it follows, that the temperature rises very rapidly in receding from the lower end of the wire, till at a small distance it becomes constant.

14. This constant temperature exceeds that given by the pyrometer by an amount varying from a seventh to a tenth.

15. The following is the table of the quantities involved in this equation:

θ.	$e^{-\mu x}$.	f.	g.
400	0.1313	325.0	25.10
600	0.1422	457.9	32.03
800	0.1521	581.8	38.21
1000	0.1603	700.0	43.85
1200	0.1678	814.3	49.11
1400	0.1736	925.8	54.00
1600	0.1788	1035.3	58.69
1800	0.1839	1143.2	63.03
2000	0.1882	1250.0	67.21
2200	0.1922	1355.8	71.23
2400	0.1957	1460.9	75.08
2600	0.1990	1565.3	78.79
2800	0.2020	1669.2	82.38
3000	0.2047	1772.6	85.87

OBSERVATORY, ARMAGH,
Nov. 8, 1848.